Separating ambiguity and volatility in cash flow simulation based volatility estimation

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Abstract

Volatility is a significant parameter both in financial and real options valuation. However, in the case of several real option projects there is no historical data available. In such cases, one alternative is to use Monte Carlo simulation on projects' cash flows for estimating volatility. An important issue that has not been taken into account with most of these volatility simulation procedures is that not only the volatility but also the value of the underlying asset is often uncertain in the beginning. Because most of the existing methods do not take this into account, they overestimate the actual volatility of the project. This paper presents a procedure that separates the underlying asset uncertainty in the beginning from the volatility and hence improves the volatility estimation.

Keywords: real options, ambiguity, volatility, cash flow simulation

JEL Classification: G31, G13

1 Introduction

Real options analysis is a framework for valuing managerial flexibility under uncertainty. It has adapted advanced methods from financial derivatives valuation and made valuation of projects with several sequential and parallel decision alternatives more accessible. Difficulty in volatility estimation, however, has hindered the success of the new valuation framework. Unlike with financial options, there is often no historical data available for volatility estimation.

Several authors have suggested different variations of applying Monte Carlo simulation on cash flow calculation to estimate the volatility. The existing cash flow simulation based volatility estimation methods are the logarithmic present value approach of Copeland & Antikarov (2001) and Herath & Park (2002), conditional logarithmic present value approach of Brandão, Dyer & Hahn (2005), two-level simulation and least-squares regression methods of Godinho (2006), and generalized risk-neutral volatility estimation over different time periods (Hull 1997). All these methods are based on the same basic idea. Monte Carlo simulation technique is applied to develop a probability distribution for the rate of return. Then, the volatility parameter σ of the underlying asset is estimated by calculating the standard deviation of the rate of return distribution. Another assumption related to all the methods is that the underlying asset value follows geometric Brownian motion (gBm). This means that the underlying asset may never have negative values and the terminal value distribution is lognormal by shape. Secondly, constant volatility for all time periods is assumed.

Although some of the existing models have either theoretical or practical weaknesses, they provide sufficiently reliable estimates in optimal conditions. However, one aspect that has not been considered carefully enough in previous research is that when doing a cash flow simulation for volatility estimation, the underlying asset value in the beginning is also likely to have uncertainty or ambiguity. When the existing methods are not applied carefully enough in such situations, these procedures provide upward biased volatility estimates.

This paper presents a volatility estimation procedure that separates the ambiguity in underlying asset value in the beginning from the uncertainty caused by volatility, and hence the method improves the volatility estimation. The procedure builds on the models and ideas presented in previous research, and therefore each of the existing methods is described briefly based on their theoretical qualities along with their strengths and weaknesses in practice. Then, the consequence of having ambiguity in the underlying asset value with the previous methods is explained. Next, a step-wise procedure for separating ambiguity and volatility in cash flow simulation based volatility estimation is presented. The solution mostly combines ideas from least-squares regression method of Godinho (2006) and generalized risk-neutral volatility estimation. After that, the procedure is illustrated with a case example adapted from Copeland & Antikarov (2001). Case also analyzes how the existing volatility simulation procedures work in case of having ambiguity in the underlying asset value. These findings are contrasted to the results of the procedure suggested in this paper with theoretical considerations and managerial implications. Finally the conclusions are presented.

2 Cash flow simulation based volatility estimation methods

Volatility is defined as a measure of the uncertainty of the return realized on an asset (Hull 1997). It is expressed as a standard deviation per year, and as a fraction of the asset's value. Volatility defines how spread the outcome distribution is expected to be. In options analysis, the standard deviation of the expected price change over a year is used to measure the volatility of the underlying asset (Howell et al., 2001). Volatility may also be regarded as the second moment of the value distribution. Volatility is probably the most difficult input parameter to estimate in real options analysis (Mun 2002), which is also the case with financial options.

There is historical data for financial derivatives which can be used for choosing and comparing different alternative stochastic models and their parameterizations to find the appropriate volatility measurement. In the case of ordinary geometric Brownian motion assumption, exponentially weighted moving average and generalized autoregressive conditional heteroscedasticity models are those applied most commonly. Also futures market information can be applied with implied volatility approach for traded assets and commodities.

However, for other investments, especially if related to R&D, there is not necessarily such information available (Lint & Pennings 1999, Newton & Pearson 1994). Therefore, volatility estimation has to be based on some other method. One alternative is to use Monte Carlo simulation for the gross present value and volatility estimation. According to Trigeorgis (1996), present value calculations may help in finding the correct volatility estimation for the project. Instead of having a marketed stock as an underlying asset, simulated gross present value is used. In this approach, forecast data for future cash values with probabilities is converted into an estimated underlying asset value and volatility (Newton & Pearson, 1994).

Logarithmic present value simulation approach (Copeland & Antikarov 2001)

The approach of consolidated volatility, *logarithmic present value approach* in terms of Mun (2002, 2003) was first introduced by Copeland and Antikarov (2001). The method relies on marketed asset disclaimer and Samuelson's proof (1965) that correctly estimated rate of return of any asset follows random walk regardless of the pattern of the cash flows. The approach is based on the idea that an investment with real options should be valued as if it was a traded asset in markets even though it would not be publicly listed. According to Copeland and Antikarov (2001), the present value of the cash flows of the project without flexibility is the best unbiased estimate of the market value of the project were it a traded asset. This is called the *marketed asset disclaimer* assumption. Therefore, simulation of cash flows should provide a reliable estimate of the investment's volatility.

According to the Copeland & Antikarov (2001) approach, Monte Carlo simulation on project's present value is used to develop a hypothetical distribution of one period returns. On each simulation trial run, the value of the future cash flows is estimated at two time periods, one for the first time period and another for the present time. The cash flows are discounted and summed to the time 0 and 1, and the following logarithmic ratio is calculated according to equation 1:

$$z = \ln\left(\frac{PV_1 + FCF_1}{PV_0}\right) \qquad (1)$$

where PV_1 means present value at time t=1, FCF_1 means free cash flow at time 1, and PV_0 project's present value at the beginning of the project at time t=0. Present value at each moment x can be calculated according to the following equation 2:

$$PV_x = \sum_{t=x+1}^{n} \frac{FCF_t}{(1+WACC)^{t-1}} \qquad (2)$$

Then, volatility σ is defined as the standard deviation of z

$$\sigma = st.dev(z) \qquad (3)$$

The model simulated is a conventional present value calculation where uncertainties related to parameters are presented as objective or subjective distributions, constants, and times series with possible correlations. After the simulation, the mean and the standard deviation of the rate of return, i.e. volatility, are calculated. Modifications to this method include duplicating the cash flows and simulating only the numerator cash flows while keeping the denominator value constant. This reduces measurement risks of auto-correlated cash flows and negative cash flows (Mun 2002), although they are still possible. Whereas simulating logarithmic cash flows gives a distribution of volatilities and therefore also a distribution of different real options values, this alternative gives a single-point estimate.

The consolidated volatility approach is analogous to stock price simulations where the theoretical stock price is the sum of all future dividend cash payments, and with real options, these cash payments are the free cash flows. The sum of free cash flows' present value at time zero is the current stock price (asset value), and at time one, the stock price in the future. The natural logarithm of the ratio of these sums is analogous to the logarithmic returns of stock prices. As stock price at time zero is known while the future stock price is uncertain, only the uncertain future stock price is simulated (Mun 2003). Of course, this does not allow for a negative outcome (or "bankruptcy") for the company, whereas operating cash flows of a single R&D project may also have negatives values.

Although the fundamental idea in Copeland & Antikarov (2001) approach is correct, it has one clear technical deficiency. The method would be appropriate volatility measure if the PV_1 were period 1's expected NPV of subsequent cash flows and this volatility would reflect the resolution of a single year's uncertainty and its impact on expectations for future cash. However, in Copeland & Antikarov's solution this PV_1 is the NPV of a particular realization of future cash flows that is generated in the simulation, and therefore the calculated standard deviation is the outcome of all future uncertainties (Smith 2005). Therefore the approach overestimates the volatility.

Herath & Park (2002) volatility estimation is very similar to Copeland & Antikarov (2001) and is based on the same equations 1 and 2. The only difference in notation is that instead of PV_0, PV_1 and FCF_1, Herath & Park use MV_0, MV_1 and A_1. However, whereas in Copeland & Antikarov (2001) only the numerator is simulated and the denominator is kept constant, Herath & Park (2002) applies simulation of both the numerator and denominator with different independent random variables: "*…both MV_0 and MV_1 are independent random variables. Therefore, a different set of random number sequences has to be generated when calculating MV_0 and MV_1*". However, this alternative has the same over-estimation problem as the original Copeland & Antikarov (2001), and it also causes additional error by having a non-constant denominator.

Conditional volatility estimation of Brandão, Dyer & Hahn (2005)

Other authors have resolved the original problem of Copeland & Antikarov's approach. Conditional volatility estimation of Brandão, Dyer & Hahn (2005), based on comments of Smith (2005), suggests an alternative where the Copeland & Antikarov (2001) simulation model is changed so that only the first year's cash flow C_1 is stochastic, and $C_2,\ldots,C_n$ are specified as expected values conditional on the outcomes of C_1. Thus, the only variability captured in PV_1 is due to the uncertainty resolved up to that point. The method works well, if the conditional future values are straightforward to calculate or estimate. Then, the standard deviation of the following equation 4 is used to estimate the volatility σ of the rate of return:

$$z = \ln\left(\frac{C_1 + PV_1(E_1(C_2),\ldots,E_1(C_n)\,|\,C_1)}{\overline{V}_0}\right) \qquad (4)$$

The deficiency with the method is that it may be hard to compute the expected future values given the values simulated in earlier periods. This is true especially for both auto- and cross-correlated input variables in cash flow simulations.

Two-level simulation of Godinho (2006)

Two-level simulation of Godinho (2006) is also based on the idea of conditionality in expected cash flows given stochastic C_1. In comparison with CVE, it works also in situations where conditional outcomes given C_1 can not be calculated analytically. Firstly, the simulation is done for the project behavior in the first year. Secondly, project behavior given the first year information is simulated for the rest of the project life cycle. Thirdly, average cash flows after the first year is used to calculate PV_1, which is then used to calculate a sample of z. Finally, volatility of z (standard deviation) is calculated. The method mostly suffers from required computation time. This is because the calculation is iterative, meaning that after each first year simulation, a large number of second stage simulation is required. Therefore, the total number of simulations is the product of first and second stage simulation runs. In practice, whereas other methods compute the volatility within a few seconds even with larger models, this procedure requires at least several minutes of computation time with present computers and algorithms. Secondly, the method requires somewhat programming skills.

Least squares regression method of Godinho (2006)

Inspired by Longstaff & Schwartz (2001), Godinho (2006) presents least squares regression method for volatility estimation. This procedure consists of two simulations. In the first simulation, the behavior of the project value and the first year information is simulated. Then, PV_1 is explained with linear regression with first year information as follows according to equation 5:

$$\widehat{PV}_1 = a_0 + b_1 X_1 + b_2 X_2 + ... + b_n X_n \qquad (5)$$

Then, in the second simulation round, volatility is calculated as the standard deviation of z

$$z = \ln\left(\frac{\widehat{PV_1}}{PV_0}\right) \tag{6}$$

Often a good and straightforward approximation is to use first year cash flow C_1 as the explaining variable with intercept term. Then, in the second simulation round, only first year cash flow is simulated, and the estimation model is used to calculate the expected value of PV_1 for calculating the sample z.

Generalized risk neutral valuation approach

There is another very effortless method for finding the volatility. It is based on the assumptions and qualities of the gBm and its lognormal underlying asset value distribution. Very similar thoughts are presented by Smith (2005) suggesting that correct parameterization for the mean value and volatility could be found by changing the volatility until the underlying asset mean and standard deviation match the simulated cash flow and its standard deviation. However, if common gBm assumptions hold, this can actually be solved analytically.

Given that PV_0 is known, and it is possible to simulate future cash flows, true or risk-neutral distribution of the cash flows in future can be simulated. As well as discounting all the cash flows to the present value, they can also be undiscounted to their future value. Because the present value of cash flows is known (PV_0), and we also know the undiscounted future value of the investment and its standard deviation, it is possible to find the volatility parameter analytically without any unnecessary additional computations and simulations. It is known that for financial assets, the asset value increases with time according to the equation 7, and that the standard deviation of the process increases according to the equation 8.

$$E(S_T) = S_0 e^{\mu T} \tag{7}$$

$$St.Dev = S_0 e^{\mu T} \sqrt{e^{\sigma^2 T} - 1} \tag{8}$$

Using equation 8, it is straightforward to calculate the annualized volatility:

$$\sigma = \sqrt{\frac{\ln\left[\left(\frac{St.Dev_e}{S_0 e^{\mu T}}\right)^2 + 1\right]}{T}} \qquad \text{(8b)}$$

The information which is required is the length of time period T for volatility estimation, value of the asset S_0 in the beginning, interest rate μ, and $St.Dev_e$ standard deviation of the asset value at the end of volatility estimation period. Interest rate μ does not change the volatility estimation as long as the same interest rate is used both in cash flow simulation and when computing the volatility. Therefore, even if the risky expected return is used in volatility estimation, the option valuation still follows risk-neutral pricing with risk-free interest rate used.

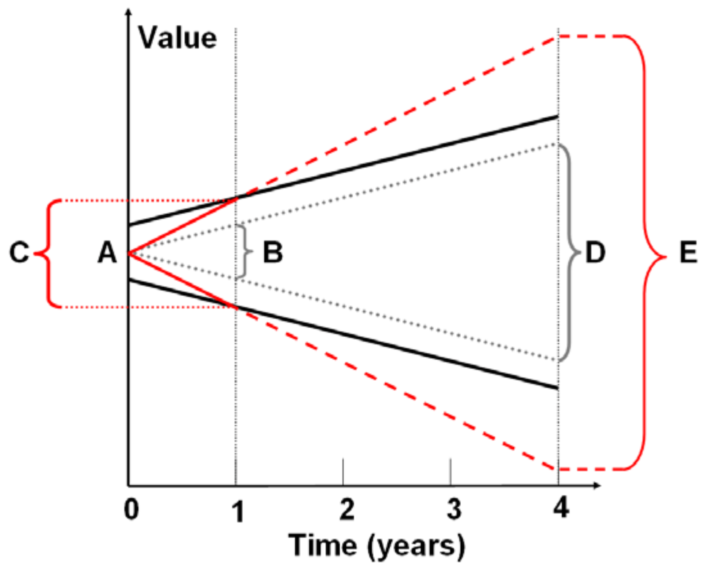
To summarize the findings of the six presented cash flow simulation based volatility methods, four of them actually provide correct results given ordinary assumptions of gBm hold, whereas two of the volatility calculation approaches, i.e. Copeland & Antikarov (2001) and Herath & Park (2002) have technical errors as suggested to be implemented by the original authors. Four other methods, conditional volatility estimation, two-level simulation, least-squares volatility simulation and generalized risk-neutral volatility calculation provide a correct measurement for volatility given ordinary assumptions of gBm and constant volatility. However, only two of the methods, least-squares regression method and generalized risk-neutral approach are sufficiently straightforward to be applied in the majority of the cases.

3 Effect of ambiguity in underlying asset value in volatility estimation

All the volatility estimation methods presented in the previous chapter suffer from one condition: they assume that the underlying asset value is constant and known in the beginning. This is true for financial options, where the underlying asset value is objectively available as stock market price. The same assumption of precisely known underlying asset value is also done with most real options cases, although no practitioner would argue that the calculated underlying asset value based on simulated cash flows case of real options would be perfectly accurate. Valuation is based on several assumptions and forecasts without perfect data available of the history. Therefore, there is always uncertainty in the underlying asset value.

There is a clear distinction in this topic between the two approaches of contingent claims analysis and dynamic programming in real options valuation (Dixit & Pindyck 1994). If there exists perfect replication and spanning by assets and contingent claims can be applied, there should not be uncertainty in terms of ambiguity in the underlying asset value in the beginning of the project. However, in the case of dynamic programming, the underlying asset is a simulated present value of the future cash flows. If the values of the simulated cash flow calculation components are not perfectly known in the beginning, also the underlying asset value is not perfectly known and has ambiguity.

This issue does not seem to be considered carefully enough when cash flow calculation and simulation is used for a) underlying asset computation and b) volatility estimation, although this uncertainty in form of ambiguity may cause serious errors in volatility estimation. The problem is that the ambiguity and the actual volatility in underlying asset value are both miscalculated as belonging to the volatility estimation, whereas only the latter one should be included. None of the authors of the previously presented methods has discussed this topic in their original papers. In case of Godinho (2006) the question is not topical because the volatility estimation is considered only for one year. Brandão et al. (2005) has a case related to the commodity industry, where the underlying asset value in the beginning is sufficiently well known, and therefore this problem does not occur in their example. On the other hand, Copeland & Antikarov (2001) do not take this into consideration in their examples, which is actually the most significant reason for the upward biased volatility estimation in their method. In earlier research the problem of upward biased volatility estimation in this method was considered to be caused mostly by the erroneous calculation of PV_1 (Smith 2005, Godinho 2006).



Graph 1: Illustration of how existing cash flow simulation based volatility estimation procedures of Copeland & Antikarov (2001), Herath & Park (2002) Brandão et al. (2005), and Godinho (2006) over-estimate the volatility because of assuming constant present value in the beginning although it is often actually uncertain with ambiguity.

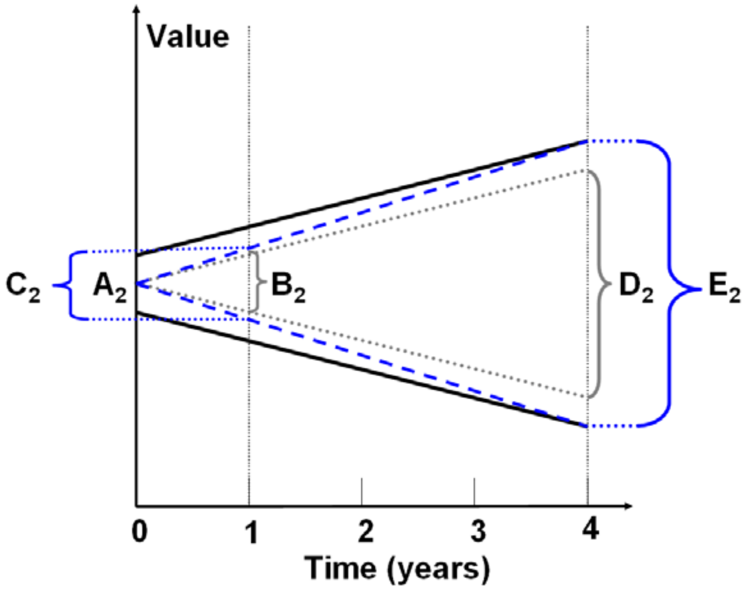
Graph 1 is used for explaining why the methods provide erroneous estimates with ambiguity in the underlying asset value in the beginning. In graph 1, the red bracket C on the left shows the deviation of the project value in year one. The black continuous lines illustrate the deviation of the underlying asset value during the four years according to the simulation forecast. This deviation is caused both by volatility and ambiguity. The dotted gray cone inside black lines is the actual deviation caused by volatility. Point A is expected project present value at time zero. Conditional volatility estimation, two-level simulation, and least-squares regression estimate assume this to be a constant number denominator.

All of these methods calculate volatility as the standard deviation of the rate of return between present value in the beginning (point A) and year one (red bracket C). The deviation in underlying asset value computation is depicted with the continuous red cone between time zero and year one. However, the actual volatility should be calculated with the same formula according to the inner gray bracket B. Because the simulation procedure does not recognize the difference between ambiguity and volatility, it combines them both into volatility measurement. Therefore, existing volatility estimations give upward biased answers.

The question is whether this matters in practice. If we are only interested in knowing the project value after year one, the decision maker does not necessarily need to know which part of the uncertainty is related to volatility and which into ambiguity. However, the situation changes if we consider a time period longer than one year. If the underlying process is assumed to follow gBm with constant volatility and the volatility is measured according to the methods discussed, the underlying asset in future deviates as presented according to the red dash line between year 1 and year 4. As the results indicate, this clearly over estimates the actual volatility. The difference between the deviation of correct process (bracket D) and the deviation of the process suggested by the methods (bracket E) can be significant.

Generalized risk-neutral valuation model when applied for a short time period has the same problem. However, the situation is very different if the method is applied for a longer time period. Graph 2 describes how the method calculates volatility in the case of a four year project. The approach is suitable in case of a longer time period, because the actual volatility increases annually, as presented by gray dotted cone D, but the ambiguity, difference between the black continuous line and the gray dotted cone, remains constant. The Proportion of ambiguity in comparison with volatility diminishes with time in comparison with other methods.

The volatility is calculated as the standard deviation of the rate of return between point A_2 and year four (blue bracket E_2). If this is annualized, the volatility in year one is the standard deviation of the rate of return between point A_2 and year one (blue bracket B_2). As the comparison of Graph 1 and Graph 2 indicates, the difference is significant, and it can be stated that generalized risk-neutral volatility estimation provides results that are much more reliable in comparison with earlier discussed methods of conditional volatility estimation, two-level simulation and least-squares regression estimation.



Graph 2: Illustration of how generalized risk-neutral volatility estimation calculates the volatility. The method provides more reliable results in the longer time period than other existing methods.

Although the generalized risk-neutral valuation may seem to be a better alternative for volatility estimation than the other methods discussed, it doesn't solve the actual problem of difference between ambiguity and volatility. The procedure only mitigates the problem of ambiguity when the time period used for the volatility estimation is longer. Secondly, even if the single constant volatility calculated with the method would provide more reliable answer than the other methods, it does not describe the actual volatility of the project over all time periods. The actual volatility is smaller than estimated by the generalized risk-neutral valuation. Secondly, the actual underlying asset price changes in time are not depicted by the process which is calculated using the estimated volatility given by generalized risk-neutral approach. A more realistic description of the process would be to have ambiguity in the beginning in underlying asset value, and then have a smaller volatility depicting the actual changes in the process when the ambiguity in the underlying asset value has diminished. This kind of modeling alternative would be both theoretically and managerially more precise and understandable by following the actual behavior of the underlying asset.

4 Procedure for separating underlying asset value ambiguity and volatility

Fortunately, the distinction between the ambiguity and volatility can be done by combining the use of the generalized risk-neutral volatility estimation and the use of the least-squares regression of Godinho (2006). Later on, the same steps are illustrated with a case example adapted from Copeland & Antikarov (2001).

In the first stage, the cash flow model is constructed. This is often a conventional present value calculation where uncertainties related to parameters are presented as subjective distributions with possible auto- and cross-correlations instead of having only single-point estimations. In the second stage, the cash flow model is simulated and the standard deviation of the terminal value distribution st.dev(PV_e) is calculated. Also a regression model is made having the time period one cash flow C_1 as an explanatory variable, which is used to explain the present value PV_1.

In the third stage, simulation is re-run, and the standard deviation of PV_1 is estimated with the regression model of the previous stage. This standard deviation is caused both by volatility and ambiguity. Now, knowing both the st.dev(PV_1) and st.dev(PV_e), it is possible to find numerically with, implied volatility technique, what constant volatility would be required to cause the increase from st.dev(PV_1) to the st.dev(PV_e) between time periods of t_1 and t_e. This can be done for example with a non-recombining binomial tree model used to model changing volatilities.

Now the volatility from time period t_1 until t_e is known. An assumption can be made that the volatility would be the same also for the time period one between time t_0 and t_1. If this holds, the change in standard deviation caused by volatility in this time period can be calculated using the equation 8. The standard deviation in time period not explained by the volatility can be considered to be caused by ambiguity. Finally, the ambiguity in the beginning t_0 can be computed by discounting the st.dev(PV_1) with risk-free interest rate.

The valuation procedure presented is very similar to the cases with changing volatility. However, it should be remembered that the existing volatility estimations used for the first year do not differ ambiguity and volatility. It may not have impact on actual decision making, but it would be better to distinguish two different phenomena form each other. The most important practical value of this procedure is that it provides a more reliable volatility estimation method than earlier cash flow methods of conditional volatility estimation (Brandão et al. 2005), generalized risk-neutral volatility estimation, and both two-level simulation and least-squares regression estimation (Godinho 2006), which do not take into account the possibility of ambiguity in the underlying asset value.

5 Case example

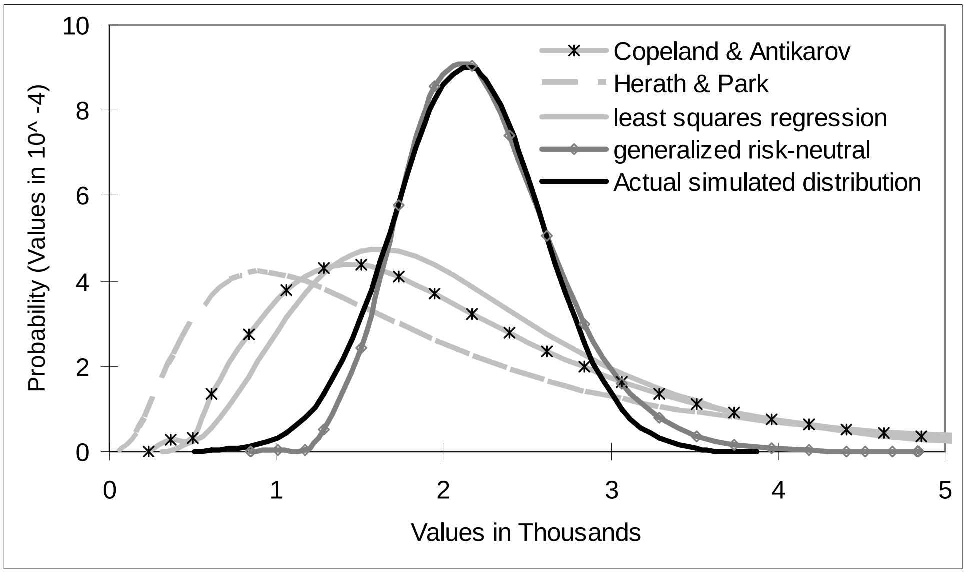
This chapter illustrates the use of the procedure presented in the previous chapter can how it can be used in practice. Although the research idea and results were originally developed in a real life case, the case presented in Copeland & Antikarov (2001, pp. 246-249) is used instead. Godinho (2006) uses the same example in his research, and therefore it is possible to compare the findings with the original results of Copeland & Antikarov (2001), Herath & Park (2002), Godinho (2006) and Brandão et al. (2005) with an example already familiar in the field of real options.

Table 1 presents the cash flow calculation of the Copeland & Antikarov (2001) case. Each Price/unit is assumed to be lognormally distributed with standard deviation of 10 % from the mean value presented in the first row. Secondly, the prices are auto-correlated with a coefficient of 90 percent. Risk-free interest rate is 5 % and weighted average cost of capital used for discounting is 12%. Therefore, present value of the cash flows is 1508.

Table 1: Cash flow calculation of Copeland & Antikarov (2001) case.

Year	1	2	3	4	5	6	7
Price/unit	10	10	9,5	9	8	7	6
Quantity	100	120	139	154	173	189	200
Variable cost/unit	6	6	5,7	5,4	4,8	4,2	3,6
Revenue	1000	1200	1321	1386	1384	1323	1200
- Variable cash costs	-600	-720	-792	-832	-830	-794	-710
- Fixed cash costs	-20	-20	-20	-20	-20	-20	-20
- Depreciation	-229	-229	-229	-229	-229	-229	-229
EBIT	151	231	279	305	305	280	241
- Taxes	-60	-92	-112	-122	-122	-112	-96
+ Depreciation	229	229	229	229	229	229	229
- Increase in working capital	-200	-40	-24	-13	0	13	24
Cash flow	120	328	373	399	412	410	398

The volatility measures provided by different alternatives are presented in the Graph 3 and Table 2. Methods of Copeland & Antikarov (2001) and Herath & Park (2002) provide the highest estimations for volatility, which can be explained both by the earlier explained deficiencies in the methods but also by the ambiguity in the underlying asset value in the beginning. Conditional volatility estimation, two-level simulation and least-squares regression estimation provide the same result, but they also suffer from the uncertain underlying asset value. Generalized risk-neutral estimation method actually provides a result which is very close to the actual simulated risk-neutral distribution.



Graph 3: Terminal value distributions with volatilities given by different methods in comparison with actual simulated terminal value distribution in Copeland & Antikarov (2001) case. Only generalized risk-neutral estimation provides a result that is very close to the actual distribution, whereas other methods have much more deviation. Results of the conditional volatility estimation of Brandão et al. (2005) and two-level simulation of Godinho (2006) are the same as the results of least squares regression method, and therefore they are not shown in the graph.

Table 2: Results provided by different volatility estimation methods in Copeland & Antikarov (2001) case. Most of the methods over-estimate the volatility because they do not take into account the ambiguity in the underlying asset value. If upward biased volatility measure is used for modelling a project with longer timespan, it will cause overestimation of the project's value.

Volatility estimation method with authors	Volatility
logarithmic present value estimation of Copeland & Antikarov (2001)	20,8 %
logarithmic present value estimation of Herath & Park (2002)	29,3 %
conditional volatility estimation of Brandão et al. (2005)	17,6 %
two-level simulation of Godinho (2006)	17,6 %
least squares regression estimation of Godinho (2006)	17,6 %
generalized risk-neutral estimation (Hull 1997)	7,7 %

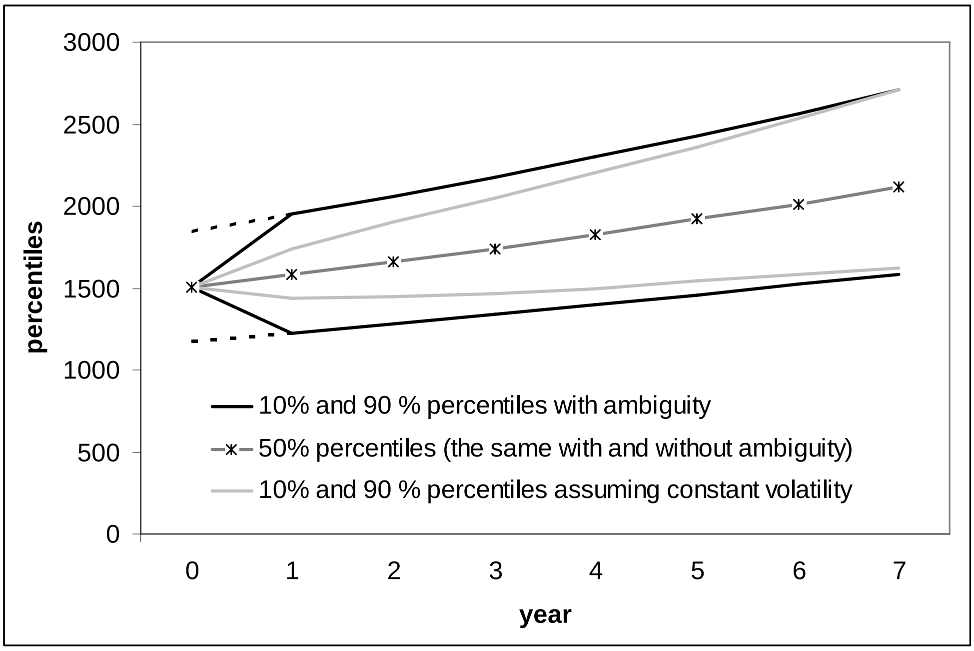
Unfortunately, the good match with the volatility fitting in comparison with the standard deviation of the terminal value distribution doesn’t yet describe how the underlying asset value changes during process. Therefore, the procedure presented in this paper is used for the volatility and ambiguity estimation.

Firstly, the cash flow simulation was constructed. The standard deviation of the terminal value distribution in risk-neutral world was computed and found to be 435. Then, the regression analysis was performed according to equation 5. The corresponding equation for the PV_1 estimation according to this was found to be PV_1 = 1032 + 5,18*C_1. Applying equation 6, volatility for the first year was 17.7 %, and the standard deviation of the PV_1 was 283.

Knowing both the st.dev(PV_1) and st.dev(PV_e), the volatility for the time period from t_1 to t_7 was calculated with to be 4.1 %. Assuming that the volatility would be the same during the whole life of the project, it was possible to calculate how much volatility causes standard deviation in PV_1 with equation 8. Volatility causes standard deviation of 65, and therefore the ambiguity causes standard deviation of 275 (subtracting volatility from the st.dev(PV_1) $\sqrt{283^2 - 65^2} \approx 275$).

6 Results and discussion

Graph 4 illustrates how the project’s PV and its deviation is assumed to change with time according to generalized risk-neutral valuation with constant volatility and according to the procedure presented here considering the effect of ambiguity. As the percentiles show, the difference is significant during the first years, but closer to the expiration both methods provide the same results. The results are very similar as in the case of the diminishing volatility. Generally, it can be stated that the effect of the ambiguity in valuation is very dependent on the timing of investment outlays. If the ambiguity is revealed before the significant investments have been made, it does not have a negative impact on the project value.



Graph 4: Illustration of how the project's present value and its deviation as a consequence of volatility is assumed to change with time according to generalized risk-neutral valuation with constant volatility (grey lines) and according to the procedure presented here considering the effect of ambiguity (black lines). The dotted black lines show what is the standard deviation during the first year caused by ambiguity. As the percentiles show, the difference is significant during the first years, but closer to the expiration both methods provide the same results.

It may be argued that the same valuation results could be found by applying a changing volatility structure and regarding all the uncertainty to be volatility. However, it should be recognized that the phenomenon that is called ambiguity in the underlying asset – not knowing the actual value of the underlying asset in the beginning – is a different kind of uncertainty compared to volatility. In the case of ambiguity, uncertainty related to a project typically is not revealed until a certain amount of time. Volatility however illustrates the continuously fluctuating uncertainty that does not dissolve during the projects lifespan.

Thus, the concepts of volatility and ambiguity should be kept theoretically apart. It may also be managerially constructive to know how much of the uncertainty is actually related to the ambiguity and how much of the uncertainty can be regarded to be caused by the volatility. The proportion of ambiguity in comparison to volatility can be considered to provide information about the precision of the valuation. Another advantage is that the procedure separates volatility that can be at least partially hedged, from ambiguity which is nearly impossible to hedge.

No research and constructed methods are without their limitations. This does not mean that the valuation itself is reliable when the ambiguity and volatility are separated mathematically. Both of these measures are based on the cash flow simulation, which however may have highly subjective estimates. The reliability of both the ambiguity and volatility estimations are very case dependent.

Another issue to bear in mind is that the procedure presented in this paper cannot tell precisely what the proportion of ambiguity and volatility is during the first time period until the ambiguity is revealed. This should not be forgotten if hedging strategies are considered in a project. However, the procedure presented in this paper reduces the risk of calculating the volatility technically wrong with cash flow simulation because unlike the existing methods it does not assume the underlying asset value to be constant.

7 Conclusions

Uncertainty in the form of ambiguity in underlying asset value causes upward biased volatility estimations with the existing cash flow simulation based models. The reason for this bias is that the methods assume that the present value of the operating cash flows is a known constant. This paper presented a step-wise procedure for separating ambiguity and volatility from each other to reduce the risk of miscalculating the volatility. The solution procedure constructed is mostly based on the least-squares regression method of Godinho (2006) and generalized risk-neutral volatility estimation. The procedure was illustrated with a case example adapted from Copeland & Antikarov (2001). The case also illustrated how the existing volatility estimation procedures work in case of having ambiguity in the underlying asset value. Separating ambiguity and volatility is essential in theory because these two forms of uncertainty are different. These differences also need to be taken into account in practice when considering possible hedging strategies.

8 References

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